Adaptive Multi-track Fitting

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Talk given at the CHEP 2000 Conference
Padova, February 2000

Outline

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Introduction

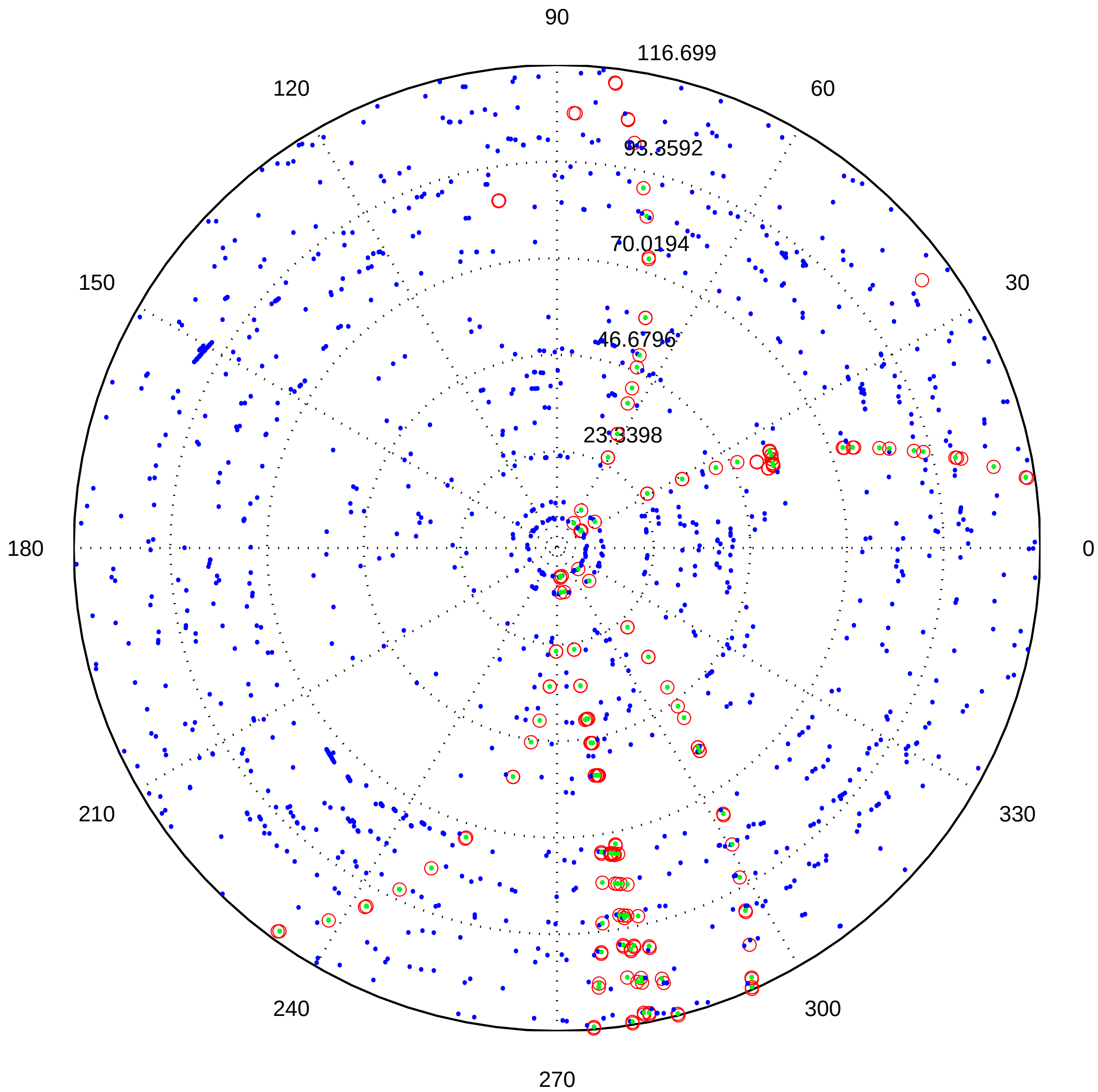
Track reconstruction at LHC will be faced by the following problems:

☞ Large background of

- low-momentum tracks
- backscattering
- electronic noise

☞ High track densities in jets

☞ In some cases ambiguous measurements (e.g. ATLAS TRT)



$H \longrightarrow \tau\tau$ event

in the CMS Inner Tracker

Consequences

- ☞ Separation between pattern recognition (track finding) and track fitting obsolete.
- ☞ Estimation of track parameters concurrently with solving the assignment problem: which hits are "signal" and which hits are "noise": "Adaptive track fitting"
- ☞ We have presented solution in the case of isolated tracks on high background: Deterministic Annealing Filter (DAF).

  The DAF relies on the EM algorithm and the Kalman filter for estimation: iterated Kalman filter with annealing, to avoid local minima.

Introduction

- ☞ The assignment problem is even more difficult if there are several tracks close to each other, for instance in a jet.
- ☞ Global competition rule could perform better than pure competition between tracks (Elastic Arms) or pure competition between hits (DAF).
- ☞ In the following we present a global competition scheme and show that it works indeed better. Further improvement by using prior information about hit/mirror hit relation.

Multi-track filters

Consider a detector layer with *m* track candidates and *n* hits.

- Track j is described by a state vector x_j (position, direction, curvature).
- Hit i is described by an observation vector y_i.
- A state and an observation which belong to the same track are coupled by the measurement equation

$$y = Hx + \varepsilon,$$

where ε is the observation error, which is assumed to be normal with zero mean and covariance matrix V. H is the measurement model.

Most important task: Compute assignment probabilities between all tracks and all hits.

Multi-track filters

As a preparation for computing the assignment probability p_{ij} between hit i and template j we set up the following matrix Φ:

$$(\Phi)_{ij} = \varphi_{ij} = \varphi(y_i; Hx_j, V_i),$$

where $\varphi(\cdot; \mu, V)$ is the multivariate normal probability density with mean vector μ and covariance matrix V.

		Tracks		
		j=1	$\dots$	j=m
Hits	i=1	φ_{11}	$\dots$	φ_{1m}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	i=n	φ_{n1}	$\dots$	φ_{nm}

We now define four methods for computing the assignment probabilities.

Method 1: Competition between hits.

There is competition between all hits for each track, but there is no competition between the tracks. This procedure is equivalent to the DAF. The assignment probabilities are computed by dividing φ_{ij} by its associated column sum plus a constant (normalization by columns):

$$p_{ij} = \frac{\varphi_{ij}}{\sum_k \varphi_{kj} + c}.$$

Method 2: Competition between tracks.

There is competition between all tracks for each hit, but there is no competition between the hits. It is equivalent to the original Elastic Arms algorithm. The assignment probabilities are computed by dividing φ_{ij} by its associated row sum plus a constant (normalization by rows):

$$p_{ij} = \frac{\varphi_{ij}}{\sum_l \varphi_{il} + c}.$$

Method 3: Global competition.

There is competition between all entries which are incompatible, i.e. belong to the same hit or to the same track. This method is proposed here for the first time.

The assignment probabilities are computed by dividing φ_{ij} by the sum of all elements in the same row and column plus a constant (normalization by columns and rows):

$$p_{ij} = \frac{\varphi_{ij}}{\sum_k \varphi_{kj} + \sum_l \varphi_{il} - \varphi_{ij} + c}.$$

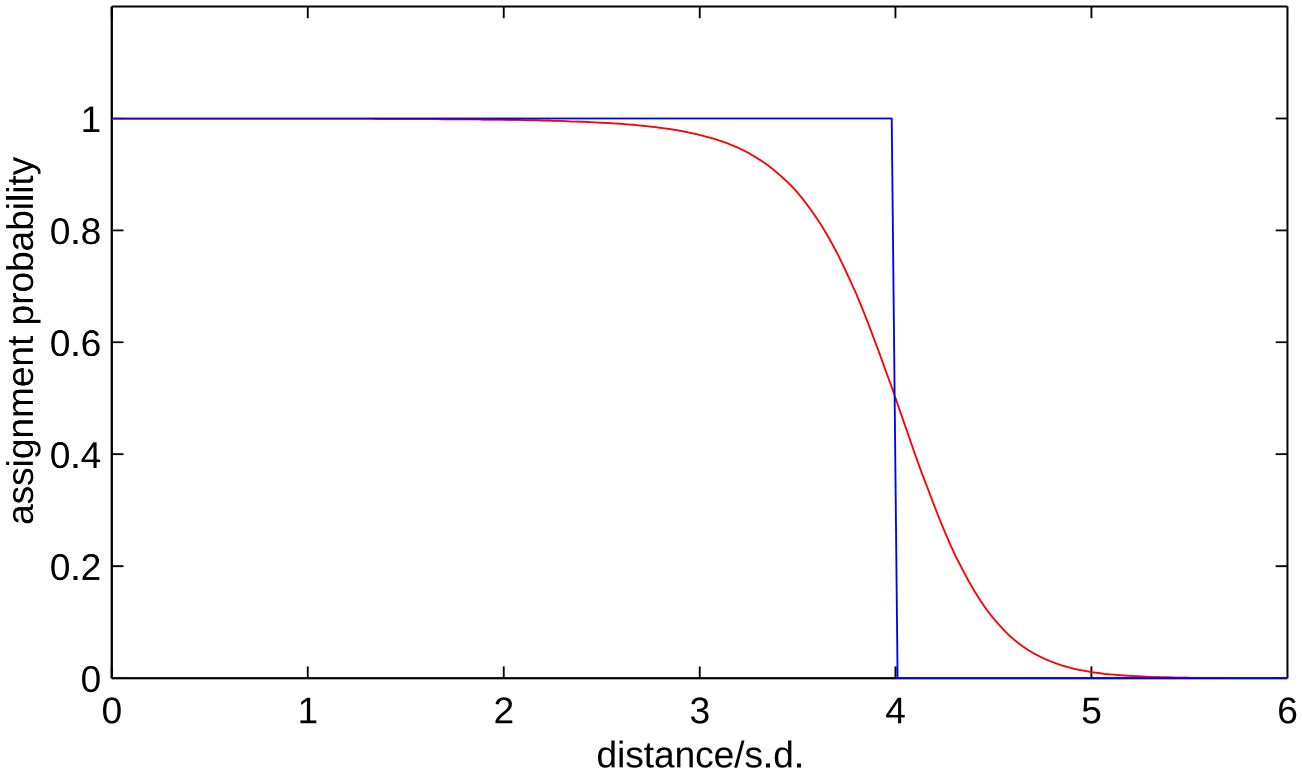
Method 4: Competition between tracks and between mirror hits.

This is a refinement of Methods 2 by adding competition between a hit and its mirror hit. It is based on Lindströms algorithm. The assignment probabilities are computed separately for each of pair of hit and mirror hit. If (i_1, i_2) is such a pair, the assignment probabilities are computed according to

$$p_{i_k j} = \frac{\varphi_{i_k j}}{\sum_l \sum_\alpha \varphi_{i_\alpha l} + c}.$$

The normalization constant is therefore the sum of all elements in the respective submatrix plus a constant. If for some reason a hit has no mirror hit it is treated according to method 2. In a detector without mirror hits methods 2 and 4 coincide.

The constant c effectively defines a cut beyond which the assignment probability quickly drops to 0.



Association probability as a function of standardized distance. Note that this function is modified if other hits or tracks are present (adaptive behaviour).

Multi-track filters

All observations with non-zero assignment probability are combined to a single observation by a weighted mean, the weights being proportional to the respective assignment probabilities:

$$z_j = \left(\sum_i p_{ij} G_i \right)^{-1} \sum p_{ij} G_i y_i, \quad \mathrm{cov}(z_j) = \left(\sum_i p_{ij} G_i \right)^{-1},$$

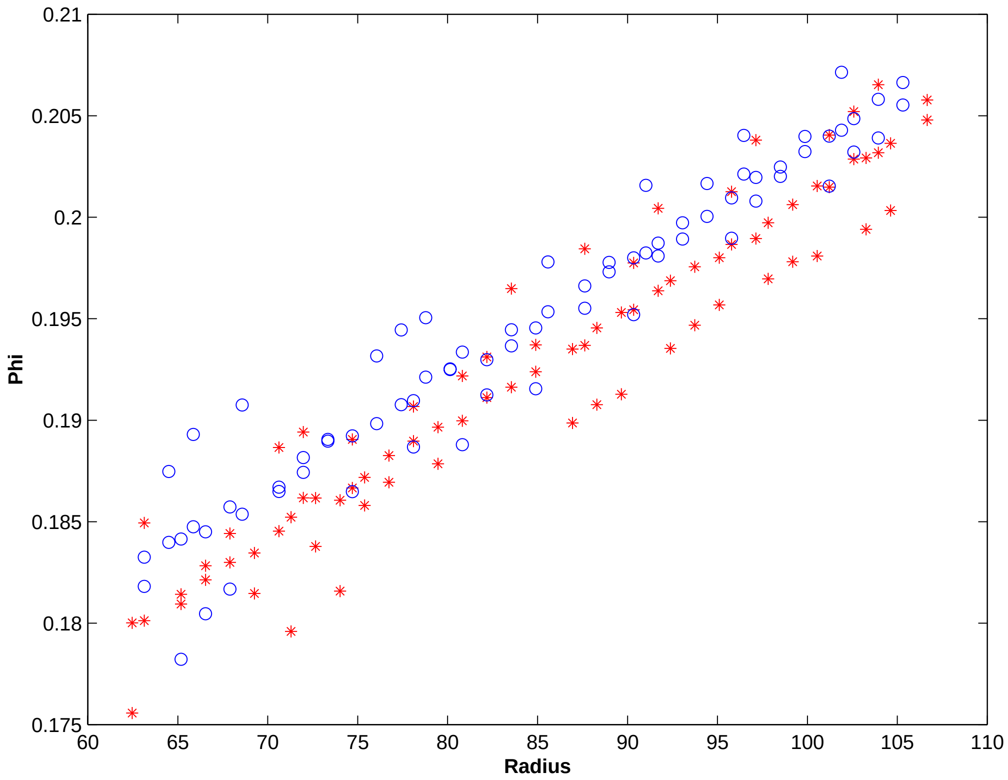
with the weight matrices G_i. This combined observation is then used in the updating step of the filter for track j.

The filter is iterated until the assignment probabilities settle to their final values. In order to avoid suboptimal solutions (local minima) the iteration is carried out with annealing, similar to the Determistic Annealing Filter.

Simulation experiments

We now present results from simulation experiments with tracks in the ATLAS Inner Detector Transition Radiation Tracker (TRT). As the TRT is made of drift tubes each hit has a corresponding mirror hit. For details of the detector see the TDR.

We have simulated a sample of 980 pairs of tracks all coming from the origin, giving 1960 tracks in total. Each track in the pair is a random perturbation of a single track. This procedure gives everything from completely separated tracks to completely overlapping tracks.



Simulation experiments

We need an initial guess of the parameters of the tracks in the pair:

- ☞ Initialize by the true values
- ☞ Initialize by the “centre of gravity”-track (cog) plus/minus one standard deviation (cog+/-).

The annealing schedule has to be chosen very carefully:

- ☞ Prevent the two templates from coming too close during the annealing.
- ☞ Choose the starting temperature not too high
- ☞ Annealing should not be too slow.
- ☞ 15 iterations at the final temperature.

Comparison of results

The precision of the estimates will be assessed by the generalized variance V of the residuals of the estimated track parameters with respect to the truth values. The generalized variances are given relative to a single-track fit with correct assignment.

Experiment 1

Mirror hits off, initialization by true values

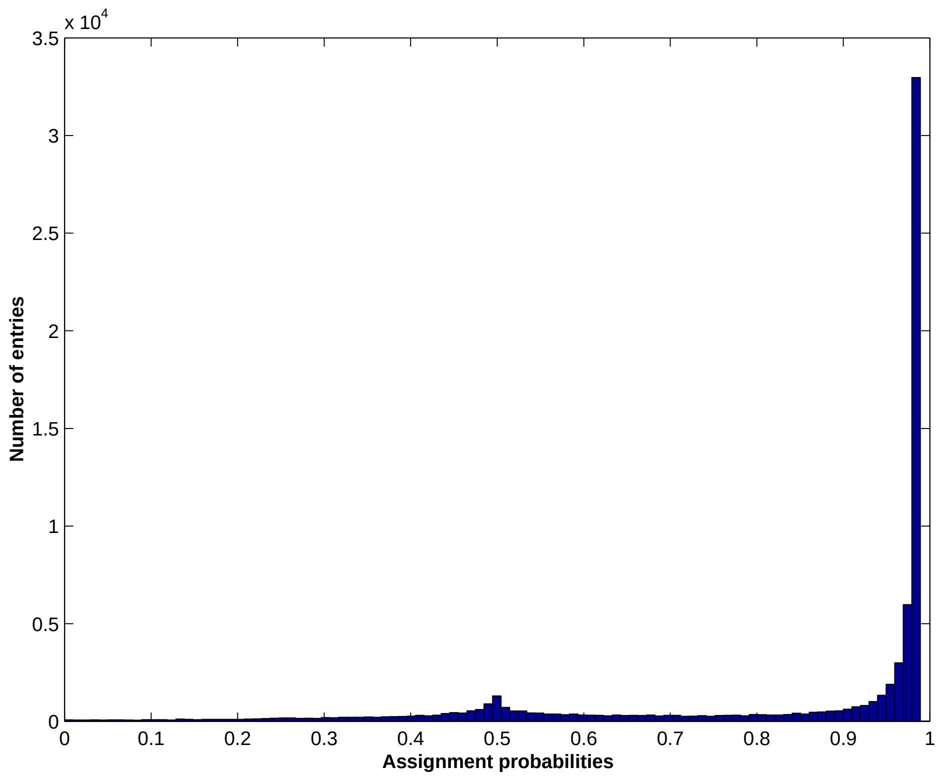
Method	1	2	3	4
V_{rel}	250	1.94	2.53	1.94

Experiment 2

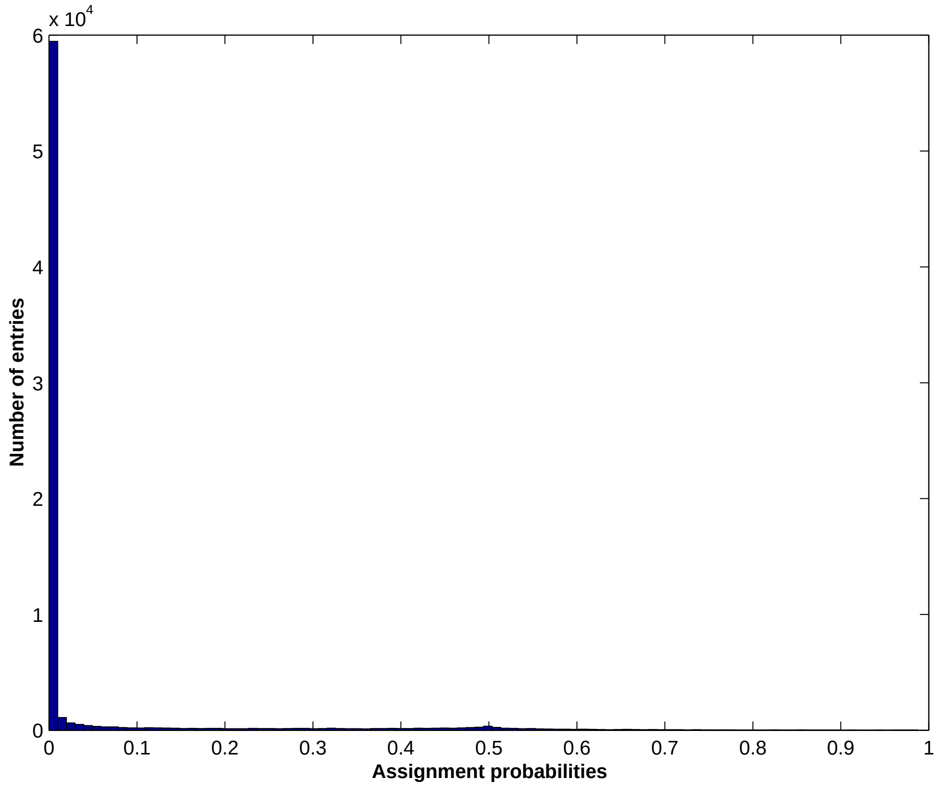
Mirror hits on, initialization by true values

Noise level	Method			
	1	2	3	4
0	281	36.2	4.52	2.84
10 %	270	58.7	5.35	4.35
20 %	388	101	6.26	7.06
30 %	358	185	7.19	9.51

Assignment probabilities of true track hits

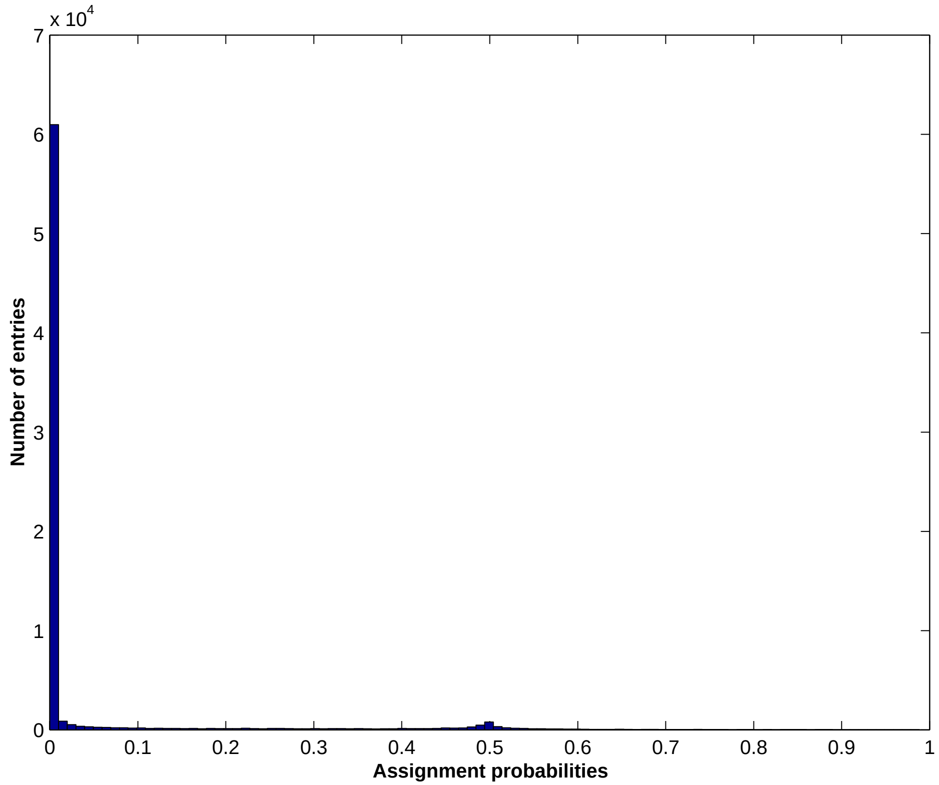


(a) True track hits

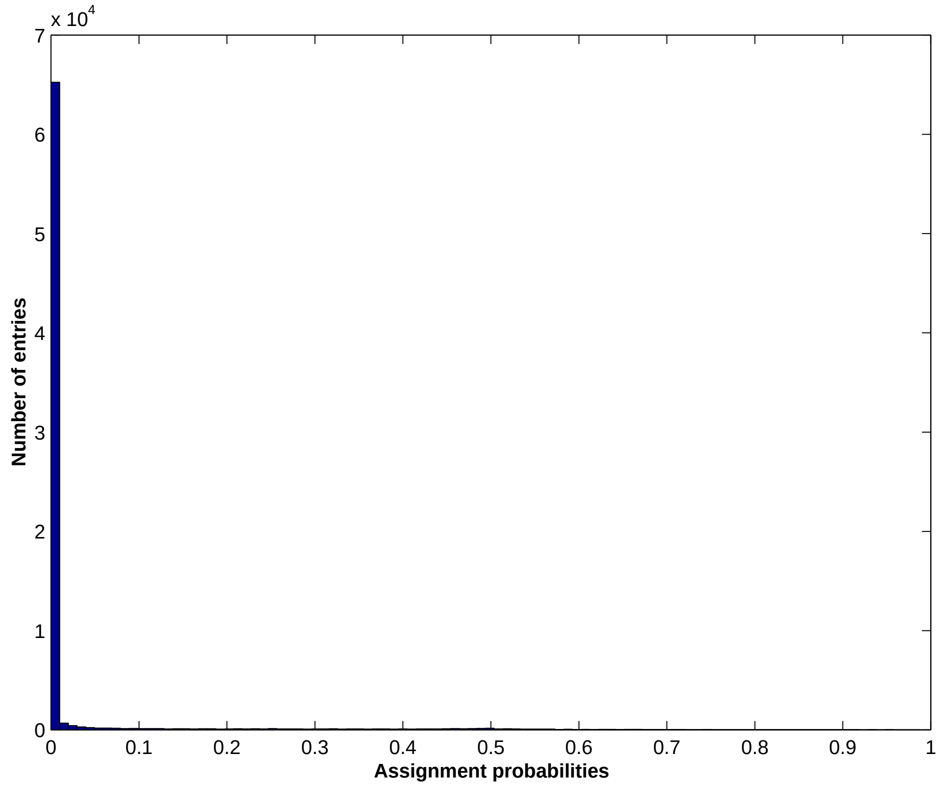


(b) Mirror hits of true tracks

Assignment probabilities of false track hits



(c) False track hits



(d) Mirror hits of false tracks

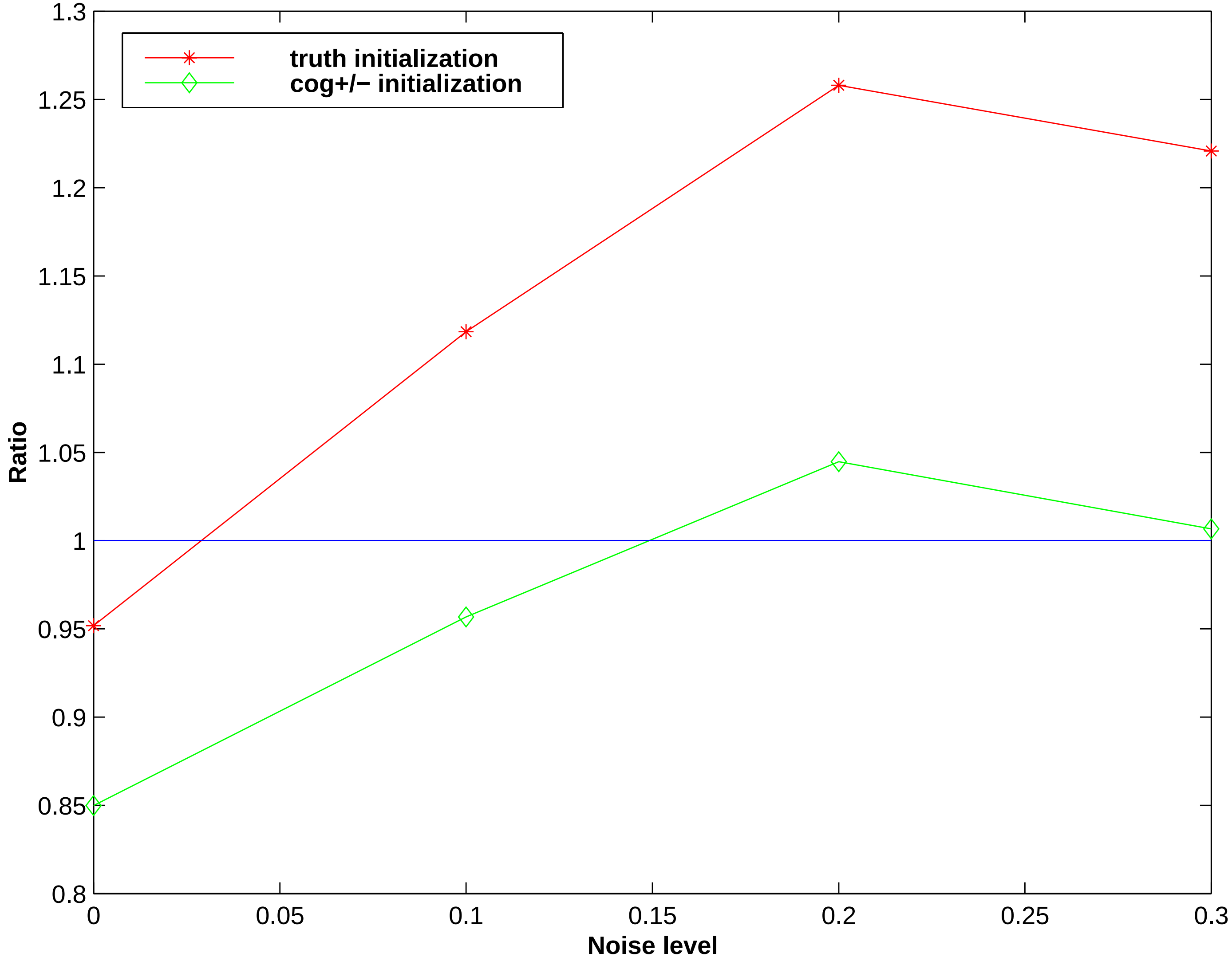
Experiment 3

Mirror hits on, initialization by cog+/-

Noise level	Method			
	1	2	3	4
0	750000	1840	72.1	11.9
10 %	753000	2700	102	21.9
20 %	968000	3530	314	40.5
30 %	980000	3730	249	43.3

Method 3 now has some problems with local minima. The generalized variance is very sensitive to these outliers. We therefore use a more robust measure of the spread: the product M of the medians of the squared track parameter residuals.

Simulation experiments



Ratios of quantity *M* (method 4 divided by method 3) versus noise level.

Conclusions and outlook

- ☞ We have introduced a new decision scheme for implementing a global competition between hits and tracks.
- ☞ We have shown that it works better than existing schemes in the presence of mirror hits and high noise levels.
- ☞ The approach is particularly interesting for detectors without mirror hits, but high background.
- ☞ We have started to extend the study to the CMS Inner Tracker.