Optimal Choice of Track Fitting Procedure for Contaminated Data in High-Accuracy Cathode Strip Chambers

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Abstract

Modern detectors of elementary particles achieve a very high accuracy of coordinate measurements (the order of tens of microns). However heavy physical backgrounds can decrease the accuracy in several orders of magnitude. Therefore one needs to choose an optimal track fitting procedure, i.e. an easy-in-use calculating approach guaranteeing the best track parameters estimation. This task is especially important for processing of contaminated data from high-accuracy cathode strip chambers. Since the traditional least squares method (LSQ) looses its optimal properties on contaminated data, some of authors made attempts to apply an LSQ modification with refitting track after rejecting more distant points. A ”straight” application of maximum likelihood method (MLM) improves the situation significantly, but leads to quite cumbersome calculating procedures. Thus the problem is to choose a user-friendly calculating procedure giving the most accurate result. We propose a robust track fitting procedure with a sub-optimal weight function. Our approach can be characterized by a mathematical simplicity, easy-calculated weights and a high-speed program realization. In our comparative study we estimate track parameters on data simulated for a cathode strip chamber applying a pure LSQ method, LSQ method with point rejecting, a ”straight” MLM and our robust technique. The results obtained demonstrate the advantages of our robust approach.

Keywords: least squares fitting, maximum likelihood method, robust fitting, cathode strip chamber, contaminated data

1 Introduction

Modern detectors of elementary particles achieve a very high accuracy of coordinate measurements (the order of tens microns). Cathode strip chambers (CSCs), i.e. six-layer multiwire proportional chambers with a strip cathode readout, are used as muon detectors in the forward region of CMS. About 10-20% of the muon hits in a CSC will be contaminated by different sources, but we consider here two of them most essential: (i) secondary electromagnetic (e.m.) particles (γ and $\bar{e}/e^+$) entering a muon detector from a calorimeter with a muon and (ii) δ-electrons produced by a muon passing through the material of a muon detector. As a result, the error distribution differs from the normal (Gaussian) distribution and tends to have long non-Gaussian ”tails”. Since the traditional least squares method (LSQ) looses its optimal properties on contaminated data, some of authors made attempts to apply an LSQ modification with refitting track after rejecting more distant points [1]. A straightforward application of maximum likelihood method (MLM) improves the situation significantly, but leads to quite cumbersome calculating procedures. Thus the problem is to choose an easy-in-use calculating procedure giving the most accurate result.

2 Mathematical formalism

Let us consider a linear regression dependence

$$x_i = \sum_{j=1}^{p} \phi_j(z_i) \cdot \theta_j \ + \ d_i \ , \quad i = 1, \ldots, N, \tag{1}$$

where $\phi_j(z)$ is a known set of p linearly independent polynomials (e.g., $1, \ z, \ \ldots z^{p-1}$); z_i - a given coordinate in the i-th detector plane; x_i - a response of the i-th detector plane (a measurement result); d_i - an accidental measurement error in this detector plane; θ_j - unknown regression parameters ($j = 1, \ldots, p$), which should be estimated by the use of data sample; N - a number of detector planes used for fitting.

We use so-called gross-error model [2] for the contaminated distribution of measurement errors d_i:

$$f(d) = (1 - \epsilon) \cdot g(d) \ + \epsilon \cdot h(d) \ \ , \tag{2}$$

where g is the Gauss distribution $g(d_i) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{d_i^2}{2 \cdot \sigma^2}}$; ϵ is a parameter of contamination; $h(d_i) = \alpha \cdot e^{-\beta \cdot |d_i|}$ - an exponential distribution.

ϵ, α and β were obtained by the parametrization of GEANT [3] results.

Applying the maximum likelihood method ($L = \prod_{i=1}^{N} f(d_i) \ \longrightarrow max$) we obtain the following system of equations

$$\sum_{i}^{N} w_i \cdot \phi_j(z_i) \cdot d_i \ + \ \beta \cdot \sigma^2 \cdot \sum_{i}^{N} \tilde{w}_i \cdot \phi_j(z_i) \cdot sign(d_i) \ = \ 0 \ , \ j = 1, \ldots, p \tag{3}$$

with optimal weights $w_i = \frac{1+c}{1+c \cdot e^{d_i^2/2 \cdot \sigma^2 - \beta \cdot |d_i|}}$ and $\tilde{w}_i \equiv 1 + c - w_i$,
where $c \equiv \frac{\sqrt{2\pi} \cdot \sigma \cdot \epsilon \cdot \alpha}{1-\epsilon}$.

Then we choose the 8th and the 4th order polynomial approximations very close to these optimal weights

$$w_p = \begin{cases} [1 \ - (\frac{d_i}{c_4 \cdot \hat{\sigma}})^4]^2 \ , & |d_i| \leq c_w \hat{\sigma} \ ; \qquad (4a) \\ [1 \ - (\frac{d_i}{c_2 \cdot \hat{\sigma}})^2]^2 \ , & c_w \hat{\sigma} < |d_i| \leq c_2 \cdot \hat{\sigma} \ ; \qquad (4b) \\ \quad 0 \ , & |d_i| > c_2 \cdot \hat{\sigma}, \qquad (4c) \end{cases} \tag{4}$$

where $\hat{\sigma}^2 = \frac{\sum w_i \cdot d_i^2}{\sum w_i}$, $c_w = 2.54$, $c_4 = 3.26$ and $c_2 = 4.19$.

The expression (4b) is in fact a bi-weight of Tukey [4]. For $w_i > 0.4$ we use the weights (4a) which are more closer to the optimal weights in this interval than bi-weights of Tukey (see Fig.1). As it can be seen from the Fig.1, the both polynomials: (4a) and (4b) coincide with the optimal weight function under the value of $w = 0.4$.

3 Comparative study

We build a Monte-Carlo (M.C.) mathematical model of a linear regression $x = az + b$ for a straight line muon track passing through 6 equidistant CSC layers. To simulate a contamination we include δ-electrons and e.m. accompaniment stochastically distributed along the muon track. The contamination parameter ϵ, the number of δ-electrons in each layer and the distance between the muon and the δ-electron are parametrised on the basis of a previous GEANT simulation [3] of muons passing

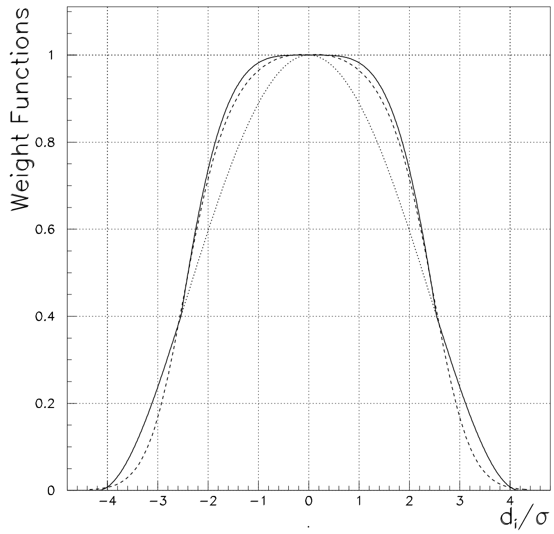

Figure 1: Weight functions in dependence of a relative deviation (W_P - a solid line; $W_{optimal}$ - a dashed line; W_{Tukey} - a dotted line).

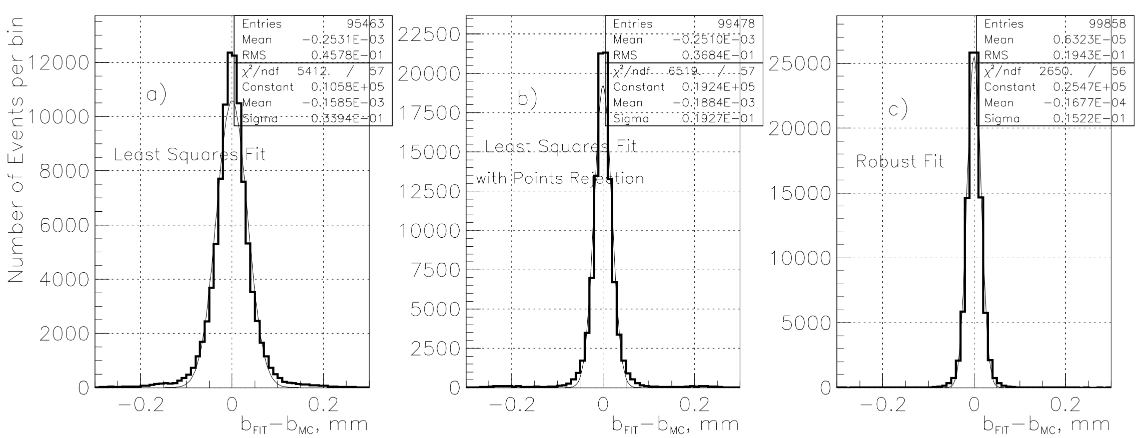

Figure 2: The distribution of deviations of straight line intercept parameters b_{FIT} from original Monte-Carlo parameters b_{MC} (fitted by a Gaussian) . Fig.2a: RMS = 0.046 mm; Fig.2b: RMS = 0.037 mm; Fig.2c: RMS = 0.019 mm . The visible decrease of a number of entries from Fig.2c to Fig.2a is caused by increasing of the lost in the corresponding methods.

through all detectors. The LSQ method with a rejection of the most distant points is described in details in [1]: after fitting of a track, the residual sum of squares is checked for the goodness-of-fit criterion; if the track does not satisfy this criterion, the most distant point from the fitting line is rejected and then the track is re-fitted.
As we can see from Fig.2, the distribution of intercept parameter deviations (db) for LSQ track fitting (Fig.2a) has longer tails than the distributions for a least squares fitting with points rejection (Fig.2b) and a robust fitting (Fig.2c). We have used a standard deviation of a Gauss distribution of measurement errors as σ = 0.044 mm). The corresponding best estimation (a lower bound) of an intercept parameter b is: $\bar{\sigma_b} = \sigma/\sqrt{N}$ = 0.018 mm. As it can be seen from Fig.2a, the value of root mean square is RMS(db) = 0.046 mm and it exceeds $\bar{\sigma_b}$ in 2.7 times. The modified LSQ with points rejection improves the situation partially but due to the "tails" RMS(db) = 0.037 mm which exceeds $\bar{\sigma_b}$ in 2 times. Only the usage of the iterative robust fitting gives a possibility to obtain the result very closed to the desired $\bar{\sigma_b}$. Track parameters obtained by the robust method (Fig.2c) have a value of RMS in 2 - 2.4 times better than the parameters obtained by the LSQ method and by the LSQ method with a rejection of distant points. For the slope parameter a we obtained a similar result. The results obtained applying a "straight" MLM method are similar to the robust fitting results but this "straight" procedure is significantly more expensive.

4 Conclusions

We propose an iterative robust track fitting procedure with a sub-optimal weight function. Our approach can be characterized by a mathematical simplicity, easy-calculated weights and a high-speed program realization. In our comparative study we estimate track parameters on data simulated for a cathode strip chamber applying a pure LSQ method, LSQ method with point rejecting, a "straight" MLM and our robust technique. The results obtained demonstrate the advantages of our robust approach, which gives the parameter estimation very close to the optimal one. Track parameters obtained by the robust method have a values of RMS in 2 - 2.4 times better than the parameters obtained by the LSQ method.

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